Homework #2 : Björn : 2025-81022

1)

a) Daily cost

- How many patients? P:= $10.2 \cdot 12 = 122.4$
- How long time do they spend on all patients? $122.4 \cdot 8 = 979.2min = 16.32h$
- How many h spent with patients per nurse? $\delta =: \frac{16.32}{2} = 8.16h$
- Expected salary for NP working t hours δ hours with patients:
  - $S = 10{,}000t + \delta 7000 \rightarrow 10{,}000 \cdot 12 + 8.16 \cdot 7000 = 177{,}120$
- Excepted cost:
  - $c = 2S = 354240$

b) Profit per day

- R=3500/visit
- How many visits per day? $P = 10.2 \cdot 12 = 122.4$
- $R = 3500 \cdot 122.4 = 428{,}400 krw/day$
- $\pi = 428{,}400 - 354{,}240 = 74{,}160 krw/day$

2)

a Porbability qustomer requesting loaner gets it

We implement as a M/M/4/4 queue using the provided excel model. I assume here nr servers s=4 and queue capacity k=4, hence M/M/4/4. In all model parpams:

- S: Nr servers =4
- K: Queue capacity = 4
- λ : Arrival rate = 3.1
- μ:service rate/server = $\frac{1}{1.2} = 0.833$

Model gives us $P_b = 10.87\%$ (probability system is full). We are thinking about the customers waiting for their service to be done as beeing in this M/M/4/4 queue, with 4 spots and 4 servers where the spots represent loaner cars. Beeing blocked in this queue means not getting a loaner.

Following we have probability system is not full: $1 - P_b = 0.8913 \approx 89\%$

If the queue is not full this means there is a loaner left.

B Avergage nr loaner cards on Wiringleys lot

From model we have average nr customers beeing served $I_p = 3.31568$

Each customer beeing served uses one car hence the number of cars not beeing used is $4 - I_p = 0.68432 cars$

C Cost per 20 days

$$c = 12n + 350 \cdot 4 + 50 \cdot q$$

n= nr loans, q=people applying but not recieving loaner

We shift model params to be per 20 days

$$\lambda = 3.1 \cdot 20 = 62$$

$$\mu = 0.833 \cdot 20 = 16.33$$

Average rate of leaving wihtout service (note recieveing loaner) $\lambda P_b = 6.74/20 days$

Evalualte c:

$$c = 12 \cdot 62 + 350 \cdot 4 + 6.74 \cdot 50 = 2{,}481 usd/20 day$$

3)

a Rose Dresses

- Current order size $Q = 30 \cdot 10 = 300$.
- Annual purchase cost = $1{,}560 \cdot 150 = \$234{,}000$.
- Orders/year = $1{,}560/300 = 5.2$, ordering cost = $5.2 \cdot 125 = \$650$.
- Holding cost = $\left(\frac{300}{2}\right) \cdot 30 = 150 \cdot 30 = \$4{,}500$.
- Total annual cost = \$234,000 + 650 + 4,500 = \$239,150.

b minimise anual cost

We want to know how many to order EOQ

Throughtput: $R = 30 * 52 = 1560 dresses/y$

Setup or order cost: $S = \$125$

Marginal anual holsing cost $H = rC = 150 \cdot 0.2 = \$30$

Cost per unit C = $\$150$

Captial cost: $r = 20\%$

Optimal order quenatity is:

$$Q_{EOQ} = \sqrt{\left(\frac{2SR}{H}\right)} = \sqrt{\left(\frac{2 \cdot 125 \cdot 1560}{30}\right)} \approx 114.02$$

- Order interval ≈ $114/30 \approx 3.80$weeks (order every ≈3.8 weeks).

- Reorder point (deterministic demand during lead time $L = 2$weeks) = $30 \cdot 2 = 60$dresses.
- Annual ordering cost = annual holding cost = about $1,710.26 each (they are equal at EOQ).
- Total annual cost = purchase cost + ordering + holding = $234{,}000 + 1{,}710.26 + 1{,}710.26 \approx \$237{,}420.53$

Optimal policy: order about 114 dresses every ≈3.8 weeks (ROP = 60). Total annual cost ≈ $237,420.5.

4)

a Level of customer scervice provided:

Lead time L=2weeks

Demand per week $N(\mu = 400, \sigma = 150)$

Reorder point R=920

During Lead time (2 weeks)

- we have $\mu_L = \mu \cdot L = 800units$
- Standard deviation $\sigma_L = \sqrt{L} \cdot \sigma = 212.13units$
- N(800,212.13)

demand during lead time : D_L

Service level provided $\mathrm{P}(D_L \leq \mathrm{ROP})$, we shift down the whole problem so it becomes standard normal, so we can look up CDF values later:

$$(D_L \leq \mathrm{ROP}) =$$

$$\frac{D_L - \mu_L}{\sigma_L} \leq \frac{(ROP - \mu_L)}{\sigma_L}$$

Hence:

$$\mathrm{P}(D_L \leq \mathrm{ROP}) =$$

$$\mathrm{P}\left(\frac{D_L - \mathrm{\mu_L}}{\mathrm{\sigma_L}} \leq \frac{(\mathrm{ROP} - \mathrm{\mu_L})}{\sigma_L}\right) =$$

$$\mathrm{P}\left(\frac{D_L - \mathrm{\mu_L}}{\mathrm{\sigma_L}} \leq 0.566\right)$$

$$\frac{D_L - \mathrm{\mu_L}}{\mathrm{\sigma_L}} : N(0,1)$$

We look up values for the standard normal CDF Φ(0.5657)≈0.7142.

≈ 71.42% service level. It is the probability mass for all the cases we get right.

b ROP for 95% service

We can look up the 95% probability for STD normal CDF which is $z \approx 1.645$ and work backwards. With the same limit as before:

$$\frac{(ROP - \mu_L)}{\sigma_L} = 1.645$$

$$ROP = 1.645\sigma_L + \mu_L = 1.645 \cdot 212.13 + 800 = 1{,}148.95units$$

5)

a. **Optimal order size at each outlet (decentralized)**

- Throughtput: $R = 4000 \cdot 50 = 200{,}000$
- Setup or order cost: $S = \$900$
- Marginal anual holsing cost $H = rC = 200 \cdot 0.2 = \$40$
- Cost per unit C = $\$200$
- Captial cost: $r = 20\%$
- Optimal order quenatity is:
- $Q_{EOQ} = \sqrt{\left(\frac{2SR}{H}\right)} = \sqrt{\frac{2 \cdot 200{,}000 \cdot 900}{40}} = 3{,}000\text{units}$
- Optimal order sze per outlet is $Q^* = 3000$ units

b. **Average time a unit spends in system (weeks)**

- Average inventory per outlet = $Q/2 = 1{,}500$units.
- Weekly demand = 4,000 units, average time in system $= 1{,}500/4{,}000 = 0.375$weeks ≈ 0.375 weeks (≈2.625 days).

c. **Effect of centralizing ordering (S increases to $1,800)**

- Decentralized total average inventory = $4 \cdot \left(\frac{Q}{2}\right) = 4 \cdot 1{,}500 = 6{,}000$units.
- Centralized EOQ for total demand $R_T = 4 \cdot 200{,}000 = 800{,}000$ and $S_c = 1{,}800$:
- $Q_c = \sqrt{\left(\frac{2SR}{H}\right)} = \sqrt{\left(\frac{2*1800*800{,}000}{40}\right)} \approx 8{,}485.28$. Total average inventory (system) = $Q_c/2 \approx 4{,}242.64$.
- Ratio central/decentral=$\frac{8{,}485.28}{3000*4}$ ≈ 0.7071 → total average inventory decreases (about 29.3% reduction).
  Conclusion: average inventory across the system will decrease under centralized ordering.

6)

- Capacity = 350 seats.
- High fare $r_H = \$780$. Low (restricted) fare $r_L = \$500$.
- High-fare demand $D \sim N(\mu = 150, \sigma{=}45)$
- Low-fare demand is high and is sold early (so we treat low fare as the “opportunity” forgone when we protect seats).

1. From slides :

$$SL^* = \frac{C_u}{C_u + C_0}$$

Whre: cost of underprotecting $C_u = r_H - r_L = 780 - 500 = 280$

Cost of overprotecting – lost low fare revenue: $C_0 = r_L = 500$

$$SL^* = \frac{280}{280 + 500} = 780280 \approx 0.35897 = 35.90\%$$

2 . Optimal protection level:

$$Q^* = Norminv(SL^*, \mu, \sigma) = Norminv(0.35897, \mu = 150, \sigma = 45) \approx 133.75$$